Ableitung der trigonometrischen Funktionen

Es gilt:

1) (sin x)´ = cos x

2) (cos x)´ = -sin x

3) Ableitung der Tangensfunktion: $f(x) = \tan x = \frac{\sin x}{\cos x}$

$$f'(x) = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{(\cos x)^2} = \frac{(\cos x)^2 + (\sin x)^2}{(\cos x)^2} = \frac{1}{(\cos x)^2}$$

$$\text{Alternative: } f'(x) = \frac{(\cos x)^2 + (\sin x)^2}{(\cos x)^2} = 1 + \frac{(\sin x)^2}{(\cos x)^2} = 1 + (\tan x)^2$$

Aufgaben:

1) $f(x) = \sin 2x$
2) $f(x) = x \cdot \sin x$
3) $f(x) = 1 - (\cos x)^2$
4) $f(x) = \frac{1}{\sin x}$
5) $f(x) = \sin x \cdot \cos x$
6) $f(x) = x^2 \cdot \cos \frac{1}{x}$
7) $f(x) = x^2 \cdot \cos x + 2x \cdot \sin x - 2 \cos x$

Lösungen:

1) $f'(x) = 2\cos 2x$

2) $f'(x) = \sin x + x \cdot \cos x$

3) $f'(x) = -2\cos x \cdot (-\sin x) = 2\cos x \cdot \sin x$

4) $f'(x) = \frac{-\cos x}{(\sin x)^2}$

5) $f'(x) = \cos x \cdot \cos x + \sin x \cdot (-\sin x) = (\cos x)^2 - (\sin x)^2$

6) $f'(x) = 2x \cdot \cos(\frac{1}{x}) + x^2 \cdot (-\sin(\frac{1}{x})) \cdot (-\frac{1}{x^2}) = 2x \cdot \cos(\frac{1}{x}) + \sin(\frac{1}{x})$

7) $f'(x) = 2x \cdot \cos x + x^2 \cdot (-\sin x) + 2\sin x + 2x \cdot \cos x + 2\sin x =$

$= 4x \cdot \cos x + 4\sin x - x^2 \cdot \sin x$