

## Aufgaben zu zusammengesetzten Bruchtermen

$$1) \frac{x}{y} : \frac{x-1}{y^2} - \frac{2xy}{x-1}$$

$$2) \left( \frac{6a^2 - 6b^2}{a^2 + 2a + 1} \cdot \frac{a^2 - 1}{2a^2 - 2ab} \right) : \frac{3a - 3}{a}$$

$$3) \left( \frac{2u+2}{3u^2-12} + \frac{1}{4-2u} \right) : \frac{2u+6}{9u+18}$$

$$4) \left( \frac{x+3}{2x+2} - \frac{x+1}{2x-2} - \frac{6}{2x^2-2} \right) \cdot \frac{4-4x^2}{3}$$

$$5) \frac{xy-y^2}{x^2-2xy+y^2} \cdot \frac{1}{x-y} - \frac{x-y}{x^2-y^2} : \frac{x+y}{x}$$

$$6) \left( \frac{-1}{x+3} - \frac{6-2x}{x^2-9} \right) : \frac{9-x^2}{x^2+6x+9}$$

## Lösungen

1)

$$\frac{x}{y} : \frac{x-1}{y^2} - \frac{2xy}{x-1} = \frac{x}{y} \cdot \frac{y^2}{x-1} - \frac{2xy}{x-1} = \frac{x \cdot y^2}{y \cdot (x-1)} - \frac{2xy}{x-1} = \frac{xy}{x-1} - \frac{2xy}{x-1} = -\frac{xy}{x-1}$$

D:  $y \neq 0$ ;  $x \neq 1$

2)

$$\begin{aligned} \left( \frac{6a^2 - 6b^2}{a^2 + 2a + 1} \cdot \frac{a^2 - 1}{2a^2 - 2ab} \right) : \frac{3a - 3}{a} &= \left( \frac{6(a^2 - b^2) \cdot (a^2 - 1)}{(a+1)^2 \cdot 2a \cdot (a-b)} \right) : \frac{3a - 3}{a} = \\ &= \left( \frac{6(a-b) \cdot (a+b) \cdot (a-1) \cdot (a+1)}{(a+1)^2 \cdot 2a \cdot (a-b)} \right) : \frac{3(a-1)}{a} = \frac{3 \cdot (a+b) \cdot (a-1)}{(a+1) \cdot a} \cdot \frac{a}{3(a-1)} = \\ &= \frac{3 \cdot (a+b) \cdot (a-1) \cdot a}{(a+1) \cdot a \cdot 3(a-1)} = \frac{a+b}{a+1} \quad \text{D: } a \neq -1; a-b \neq 0; a \neq 0; a \neq 1 \end{aligned}$$

3)

$$\begin{aligned} \left( \frac{2u+2}{3u^2-12} + \frac{1}{4-2u} \right) : \frac{2u+6}{9u+18} &= \left( \frac{2(u+1)}{3(u^2-4)} + \frac{1}{2(2-u)} \right) : \frac{2(u+3)}{9(u+2)} = \\ &= \left( \frac{2(u+1)}{3(u-2) \cdot (u+2)} + \frac{1}{2 \cdot (-1) \cdot (u-2)} \right) \cdot \frac{9(u+2)}{2(u+3)} = \\ &= \frac{2(u+1) \cdot 2 \cdot (-1) + 1 \cdot 3 \cdot (u+2)}{3 \cdot (u-2) \cdot (u+2) \cdot 2 \cdot (-1)} \cdot \frac{9(u+2)}{2(u+3)} = \frac{-4u-4+3u+6}{3 \cdot (u-2) \cdot (u+2) \cdot 2 \cdot (-1)} \cdot \frac{9(u+2)}{2(u+3)} = \\ &= \frac{(-u+2) \cdot 9 \cdot (u+2)}{3 \cdot (u-2) \cdot (u+2) \cdot 2 \cdot (-1) \cdot 2 \cdot (u+3)} = \frac{(-1) \cdot (u-2) \cdot 9 \cdot (u+2)}{3 \cdot (u-2) \cdot (u+2) \cdot 2 \cdot (-1) \cdot 2 \cdot (u+3)} = \frac{3}{4(u+3)} \end{aligned}$$

D =  $\mathbb{R} \setminus \{-3; -2; 2\}$

4)

$$\begin{aligned}
 & \left( \frac{x+3}{2x+2} - \frac{x+1}{2x-2} - \frac{6}{2x^2-2} \right) \cdot \frac{4-4x^2}{3} = \\
 & = \left( \frac{(x+3) \cdot (x-1)}{2 \cdot (x+1) \cdot (x-1)} - \frac{(x+1) \cdot (x+1)}{2 \cdot (x+1) \cdot (x-1)} - \frac{6}{2 \cdot (x+1) \cdot (x-1)} \right) \cdot \frac{4-4x^2}{3} = \\
 & = \frac{x^2 - x + 3x - 3 - (x^2 + 2x + 1) - 6}{2 \cdot (x+1) \cdot (x-1)} \cdot \frac{4 \cdot (1-x) \cdot (1+x)}{3} = \\
 & = \frac{-10}{2 \cdot (x+1) \cdot (x-1)} \cdot \frac{4 \cdot (1-x) \cdot (1+x)}{3} = \frac{-10 \cdot 4 \cdot (1-x) \cdot (1+x)}{2 \cdot (x+1) \cdot (x-1) \cdot 3} = \\
 & = \frac{-10 \cdot 4 \cdot (-1) \cdot (x-1) \cdot (1+x)}{2 \cdot (x+1) \cdot (x-1) \cdot 3} = \frac{20}{3} \quad D = \mathbb{R} \setminus \{-1; 1\}
 \end{aligned}$$

5)

$$\begin{aligned}
 & \frac{xy-y^2}{x^2-2xy+y^2} \cdot \frac{1}{x-y} - \frac{x-y}{x^2-y^2} \cdot \frac{x+y}{x} = \frac{y \cdot (x-y) \cdot 1}{(x-y)^2 \cdot (x-y)} - \frac{(x-y)}{(x-y)(x+y)} \cdot \frac{x}{x+y} = \\
 & = \frac{y}{(x-y)^2} - \frac{(x-y) \cdot x}{(x-y)(x+y) \cdot (x+y)} = \frac{y}{(x-y)^2} - \frac{x}{(x+y)^2} = \frac{y \cdot (x+y)^2 - x \cdot (x-y)^2}{(x-y)^2 \cdot (x+y)^2} = \\
 & = \frac{x^2y + 2xy^2 + y^3 - x^3 + 2x^2y - xy^2}{(x-y)^2(x+y)^2} = \frac{-x^3 + 3x^2y + xy^2 + y^3}{(x-y)^2(x+y)^2}
 \end{aligned}$$

$$D: x \neq 0; x+y \neq 0; x-y \neq 0$$

6)

$$\begin{aligned}
 & \left( \frac{-1}{x+3} - \frac{6-2x}{x^2-9} \right) \cdot \frac{9-x^2}{x^2+6x+9} = \left( \frac{-1 \cdot (x-3) - (6-2x)}{(x+3) \cdot (x-3)} \right) \cdot \frac{(3-x) \cdot (3+x)}{(x+3)^2} = \\
 & \frac{-x+3-6+2x}{(x+3) \cdot (x-3)} \cdot \frac{3-x}{3+x} = \frac{x-3}{(x+3) \cdot (x-3)} \cdot \frac{3+x}{3-x} = \frac{(x-3) \cdot (3+x)}{(x+3) \cdot (x-3) \cdot (3-x)} = \frac{1}{3-x} \\
 & D = \mathbb{R} \setminus \{-3; 3\}
 \end{aligned}$$