

5.

$$\begin{array}{l} \text{(I)} \\ \text{(II)} \\ \text{(III)} \end{array} \left(\begin{array}{ccc|c} 3 & 1 & 0 & 0 \\ 4 & -4 & 1+t & 0 \\ -1 & 1 & 4+t & 0 \end{array} \right) \quad \begin{array}{l} \text{(I)} \\ \text{(II)} \\ \text{(III)} \end{array} \left(\begin{array}{ccc|c} 3 & 1 & 0 & 0 \\ 0 & -16 & 3+3t & 0 \\ 0 & 4 & 12+3t & 0 \end{array} \right) \quad \begin{array}{l} \text{(I)} \\ \text{(II)} \\ \text{(III)} \end{array} \left(\begin{array}{ccc|c} 3 & 1 & 0 & 0 \\ 0 & -16 & 3+3t & 0 \\ 0 & 0 & -204-60t & 0 \end{array} \right)$$

$$-204 - 60t = 0 \Rightarrow t = -3, 4$$

$t = -3, 4$: Vektoren sind linear abhängig

$t \in \mathbb{R} \setminus \{-3, 4\}$: Vektoren sind linear unabhängig

6.1

$$\lambda_1 \vec{u} + \lambda_2 \vec{v} = \vec{0} \Rightarrow \lambda_1 (\vec{a} - \vec{b}) + \lambda_2 (\vec{a} + \vec{b}) = \vec{0}$$

$$(\lambda_1 + \lambda_2) \vec{a} + (\lambda_2 - \lambda_1) \vec{b} = \vec{0}$$

$$\text{(I)} \quad \lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_2 = -\lambda_1$$

$$\text{(II)} \quad \lambda_2 - \lambda_1 = 0 \Rightarrow \lambda_1 = 0 \Rightarrow \lambda_2 = 0$$

$\Rightarrow \vec{u}$ und \vec{v} sind linear unabhängig

6.2

$$\lambda_1 \vec{u} + \lambda_2 \vec{v} = \vec{0} \Rightarrow \lambda_1 \left(\frac{1}{2} \vec{a} + \frac{1}{2} \vec{b} \right) + \lambda_2 (\vec{b} - \vec{a}) = \vec{0}$$

$$\left(\frac{1}{2} \lambda_1 - \lambda_2 \right) \vec{a} + \left(\frac{1}{2} \lambda_1 + \lambda_2 \right) \vec{b} = \vec{0}$$

$$\text{(I)} \quad \frac{1}{2} \lambda_1 - \lambda_2 = 0 \Rightarrow \lambda_2 = \frac{1}{2} \lambda_1$$

$$\text{(II)} \quad \frac{1}{2} \lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_1 = 0 \Rightarrow \lambda_2 = 0$$

$\Rightarrow \vec{u}$ und \vec{v} sind linear unabhängig

7.1

$$\overline{EG} = \overline{EA} + \overline{AG} = -\vec{c} + \frac{1}{2} \vec{a}$$

$$\overline{MS} = \overline{MC} + \overline{CS} = \frac{1}{2} \overline{AC} + \frac{2}{3} \overline{CM_2} \quad (M_2 \text{ Mittelpunkt von } [EB])$$

$$\overline{MS} = \frac{1}{2} (\vec{a} + \vec{b}) + \frac{2}{3} \left[-\vec{b} + \frac{1}{2} (-\vec{a} + \vec{c}) \right] = \frac{1}{6} \vec{a} - \frac{1}{6} \vec{b} + \frac{1}{3} \vec{c}$$

7.2