

Lineare Gleichungssysteme mit vier Unbekannten

Beispiele:

1) Bestimmen Sie die Lösung des folgenden linearen Gleichungssystems.

$$(I) \quad x_1 + 3x_3 - 2x_4 = 11$$

$$(II) \quad 3x_1 - 2x_2 + x_3 = 7$$

$$(III) \quad -x_1 + 4x_2 + 2x_3 + 2x_4 = -5$$

$$(IV) \quad 3x_1 - 3x_2 - 5x_3 + x_4 = -6$$

2) Bestimmen Sie die Lösung des folgenden linearen Gleichungssystems.

$$(I) \quad x_1 - x_2 + 2x_3 - 2x_4 = -9$$

$$(II) \quad 3x_2 + 4x_3 - x_4 = 3$$

$$(III) \quad 3x_1 - 2x_2 + 3x_3 = -9$$

$$(IV) \quad 4x_1 + 2x_2 - x_3 + x_4 = 9$$

Lösungen:

1)

$$\begin{array}{l} (I) \left(\begin{array}{cccc|c} 1 & 0 & 3 & -2 & 11 \\ 3 & -2 & 1 & 0 & 7 \\ -1 & 4 & 2 & 2 & -5 \\ 3 & -3 & -5 & 1 & -6 \end{array} \right) \\ (II) \left(\begin{array}{cccc|c} 3 & -2 & 1 & 0 & 7 \\ -1 & 4 & 2 & 2 & -5 \\ 3 & -3 & -5 & 1 & -6 \end{array} \right) \\ (III) \left(\begin{array}{cccc|c} -1 & 4 & 2 & 2 & -5 \\ 3 & -3 & -5 & 1 & -6 \end{array} \right) \\ (IV) \left(\begin{array}{cccc|c} 3 & -3 & -5 & 1 & -6 \end{array} \right) \end{array} \rightarrow \begin{array}{l} (I) \left(\begin{array}{cccc|c} 1 & 0 & 3 & -2 & 11 \\ 0 & -2 & -8 & 6 & -26 \\ 0 & 4 & 5 & 0 & 6 \\ 0 & -3 & -14 & 7 & -39 \end{array} \right) \\ (II) \left(\begin{array}{cccc|c} 0 & -2 & -8 & 6 & -26 \\ 0 & 4 & 5 & 0 & 6 \\ 0 & -3 & -14 & 7 & -39 \end{array} \right) \\ (III) \left(\begin{array}{cccc|c} 0 & 4 & 5 & 0 & 6 \\ 0 & -3 & -14 & 7 & -39 \end{array} \right) \\ (IV) \left(\begin{array}{cccc|c} 0 & -3 & -14 & 7 & -39 \end{array} \right) \end{array}$$

$$\rightarrow \begin{array}{l} (I) \left(\begin{array}{cccc|c} 1 & 0 & 3 & -2 & 11 \\ 0 & -2 & -8 & 6 & -26 \\ 0 & 0 & 22 & -24 & 92 \\ 0 & 0 & 4 & 4 & 0 \end{array} \right) \\ (II) \left(\begin{array}{cccc|c} 0 & -2 & -8 & 6 & -26 \\ 0 & 0 & 22 & -24 & 92 \\ 0 & 0 & 4 & 4 & 0 \end{array} \right) \\ (III) \left(\begin{array}{cccc|c} 0 & 0 & 22 & -24 & 92 \\ 0 & 0 & 4 & 4 & 0 \end{array} \right) \\ (IV) \left(\begin{array}{cccc|c} 0 & 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 184 & -368 \end{array} \right) \end{array}$$

$$(IV) \Rightarrow 184x_4 = -368 \Rightarrow x_4 = -2$$

$$(III) \Rightarrow 22x_3 - 24x_4 = 92 \Rightarrow x_3 = 2$$

$$(II) \Rightarrow -2x_2 - 8x_3 + 6x_4 = -26 \Rightarrow x_2 = -1$$

$$(I) \Rightarrow x_1 + 3x_3 - 2x_4 = 11 \Rightarrow x_1 = 1$$

$\Rightarrow (1 / -1 / 2 / -2)$ ist Lösung des LGS

2)

$$\begin{array}{l} (I) \left(\begin{array}{cccc|c} 1 & -1 & 2 & -2 & -9 \\ 0 & 3 & 4 & -1 & 3 \\ 3 & -2 & 3 & 0 & -9 \\ 4 & 2 & -1 & 1 & 9 \end{array} \right) \\ (II) \left(\begin{array}{cccc|c} 0 & 3 & 4 & -1 & 3 \\ 3 & -2 & 3 & 0 & -9 \\ 4 & 2 & -1 & 1 & 9 \end{array} \right) \\ (III) \left(\begin{array}{cccc|c} 3 & -2 & 3 & 0 & -9 \\ 4 & 2 & -1 & 1 & 9 \end{array} \right) \\ (IV) \left(\begin{array}{cccc|c} 4 & 2 & -1 & 1 & 9 \end{array} \right) \end{array} \rightarrow \begin{array}{l} (I) \left(\begin{array}{cccc|c} 1 & -1 & 2 & -2 & -9 \\ 0 & 3 & 4 & -1 & 3 \\ 0 & 1 & -3 & 6 & 18 \\ 0 & 6 & -9 & 9 & 45 \end{array} \right) \\ (II) \left(\begin{array}{cccc|c} 0 & 3 & 4 & -1 & 3 \\ 0 & 1 & -3 & 6 & 18 \\ 0 & 6 & -9 & 9 & 45 \end{array} \right) \\ (III) \left(\begin{array}{cccc|c} 0 & 1 & -3 & 6 & 18 \\ 0 & 6 & -9 & 9 & 45 \end{array} \right) \\ (IV) \left(\begin{array}{cccc|c} 0 & 6 & -9 & 9 & 45 \end{array} \right) \end{array}$$

$$\rightarrow \begin{array}{l} (I) \left(\begin{array}{cccc|c} 1 & -1 & 2 & -2 & -9 \\ 0 & 3 & 4 & -1 & 3 \\ 0 & 0 & -13 & 19 & 51 \\ 0 & 0 & -51 & 33 & 117 \end{array} \right) \\ (II) \left(\begin{array}{cccc|c} 0 & 3 & 4 & -1 & 3 \\ 0 & 0 & -13 & 19 & 51 \\ 0 & 0 & -51 & 33 & 117 \end{array} \right) \\ (III) \left(\begin{array}{cccc|c} 0 & 0 & -13 & 19 & 51 \\ 0 & 0 & -51 & 33 & 117 \end{array} \right) \\ (IV) \left(\begin{array}{cccc|c} 0 & 0 & 0 & 540 & 1080 \end{array} \right) \end{array}$$

$$(IV) \Rightarrow 540x_4 = 1080 \Rightarrow x_4 = 2$$

$$(III) \Rightarrow -13x_3 + 19x_4 = 51 \Rightarrow x_3 = -1$$

$$(II) \Rightarrow 3x_2 + 4x_3 - x_4 = 3 \Rightarrow x_2 = 3$$

$$(I) \Rightarrow x_1 - x_2 + 2x_3 - 2x_4 = -9 \Rightarrow x_1 = 0$$

$\Rightarrow (0 / 3 / -1 / 2)$ ist Lösung des LGS